

# Solving k-SAT on quantum machines

Alain Sarlette — QUANTIC — inria Paris

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# Outline

I. Optimization / classical problem

II. Quantum annealing methods

III. Harnessing quantum dissipation

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## I. Optimization / classical problem

k-SAT : computational approach

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Tutorial

## II. Quantum annealing methods

standard quantum annealing

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new quantum annealing

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New developments

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quantum Zeno dragging

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Main result

k-SAT combinatorial optimization paradigm

$$x_j \text{ in } \{0, 1\} = \{\text{true}, \text{false}\} \text{ for } j=1, 2, \dots, n$$

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Can we get « true » as an output of :

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Example: yes, take e.g.

$x_1=x_2=\text{false}$

$x_3=\text{true}$

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NP – complete : Hard to solve for some instances ( $\sim 2^{\alpha n}$ )

Equivalent to whole class of (practical) problems

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(best current guarantee)

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Achieving quantum speedup with Grover search

= by using calls to  $x \rightarrow G(x)=1$  or  $G(x)=0$  in quantum superposition,  
finds the  $M$  values of  $x$  satisfying  $G(x)=1$ , among  $2^n$ , in runtime  $\sqrt{2^n/M}$

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- backtracking trees: Grover-inspired quantum walk procedure  $\rightarrow T_{new} \sim \sqrt{T}$

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k-SAT cost functions could have more **structure**

e.g.  $G(x)$  = number of unsatisfied clauses

Open question: quantum algebraic acceleration exploiting this?

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Hidden in this procedure: rising cost of uncomputing

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Adiabatic propagation theorem:

$|\psi(t)\rangle$  remains guaranteed close to the ground state of  $H(t)$  if

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→ runtime  $T \sim \|H'\| / \Delta^2$

$H_1$  induces avoided crossings

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! moreover, building this  $G(x)$  from local interactions may be hard



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! also more natural implementation, local couplings

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- Gap amplification (details later): 1 ancilla level per clause gives  $\Delta_{\text{new}} \sim \sqrt{\Delta_{\text{old}}}$   
but now with only guarantee  $T \sim \frac{1}{\Delta_{\text{new}}^2} \sim 1/\Delta_{\text{old}}$

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# A different take on $H(t)$ in Quantum Annealing

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$\{ |\mathbf{x}(0)\rangle = |++\dots+\rangle \text{ for all } \mathbf{x} \}$ :

$H(0)$ : trivial Hamiltonian,  
highly degenerate

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$$|\mathbf{x}(s)\rangle = \cos(s\pi/4) |+\rangle \pm \sin(s\pi/4) |-\rangle$$

on each qubit

$|0_{\text{logic}}\rangle$

$|1_{\text{logic}}\rangle$

almost degenerate for  $s \ll 1$

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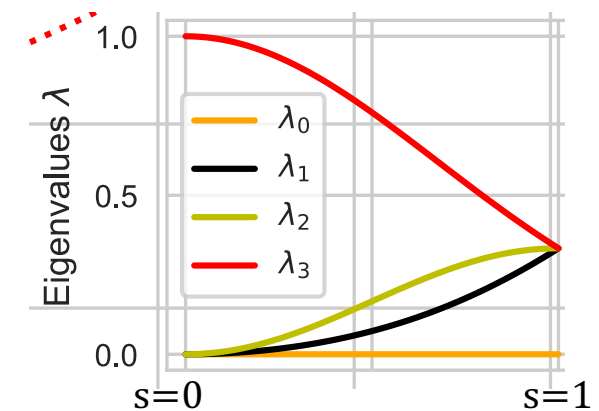
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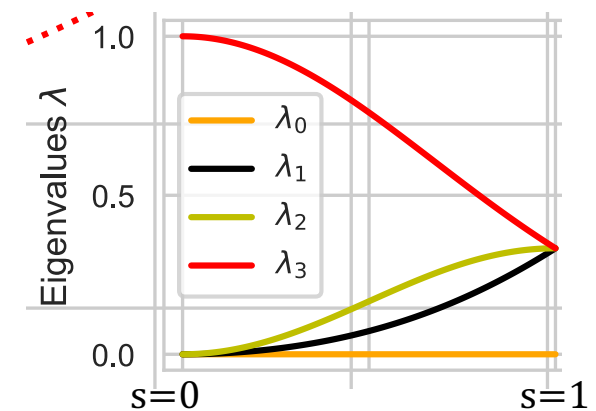
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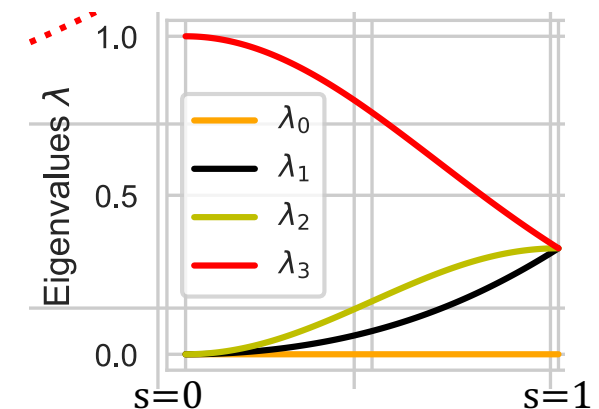


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 $= |\langle 011 \dots 101_{\text{logic}}(s_1) | \text{---...-} \rangle| \sim \cos(s_1)^n$

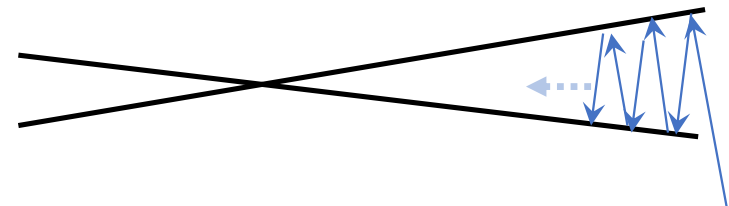
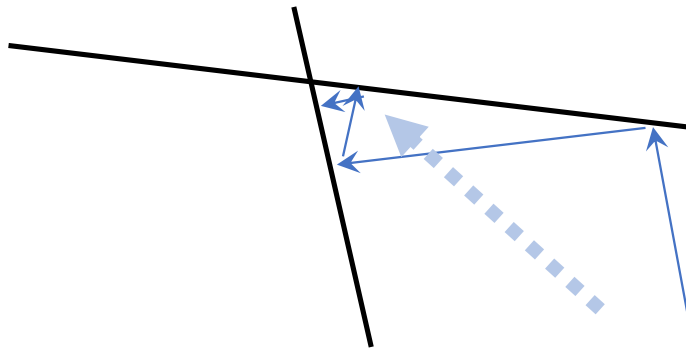
Runtime  $T$  follows this tradeoff in the choice of  $s_1$



# (weak) relation to alternated projections method

To find intersection between subspaces: project alternately onto each

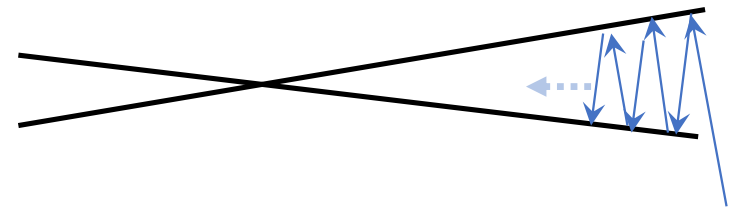
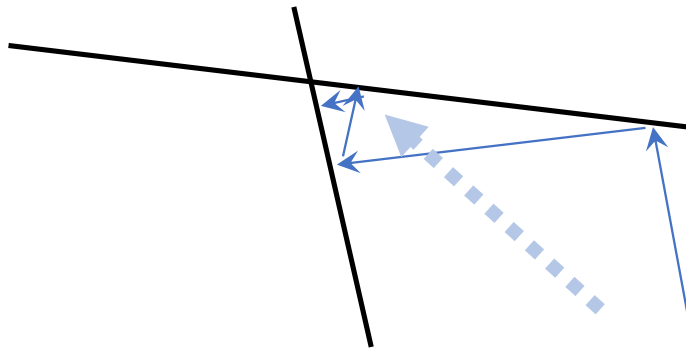
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To find intersection between subspaces: project alternately onto each

- Almost parallel subspaces imply slow convergence
- Here, track slowly opening intersection, until it encodes k-SAT solution

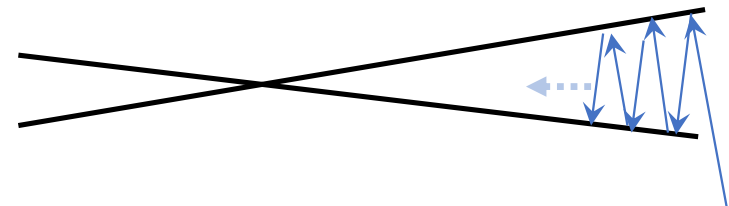
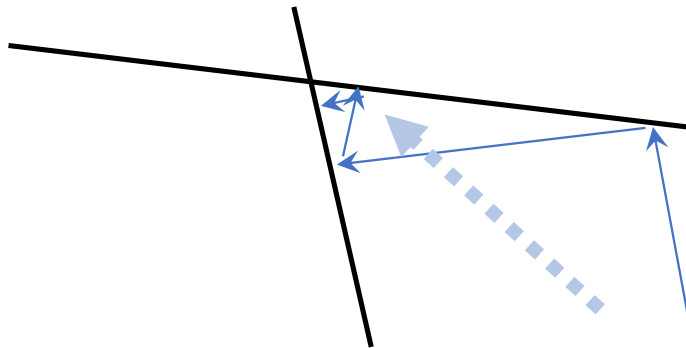




# (weak) relation to alternated projections method

Detectability Lemma: Let  $P_1, P_2, \dots, P_m$  quasi-local with  $\leq g$  pairwise overlaps

$$\text{then } \|\prod_k (I - P_k) |\psi_\perp\rangle\|^2 \leq 1 / (1 + \Delta/g^2)$$



Recap: k-SAT annealing with t-dependent basis

$$|x(t)\rangle = \cos(s_t \pi/4) |+\rangle \pm \sin(s_t \pi/4) |-\rangle \quad \text{on each qubit}$$

true  
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$$H = \sum_m P_{(\text{failing clause } m)}(x(t))$$

- Initial loss of fidelity: sub-exponential with  $t_1 \sim 1/\sqrt{n}$
- With gap amplification: runtime  $T \sim 1/\sqrt{\Delta(s_1)}^2 \sim 1/\Delta(s_1)$
- Bound on  $\Delta(s_1)$ : still work in progress...

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Equivalent Lindbladian:

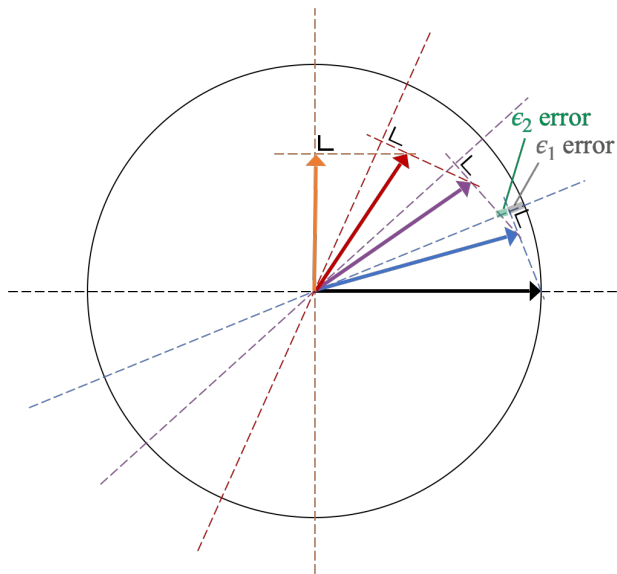
$$\frac{d}{dt} \rho = \frac{1}{m} \sum_m P_m \rho P_m - \frac{1}{2} P_m \rho - \frac{1}{2} \rho P_m$$



Straightforward analysis with discrete steps  $x(t)$

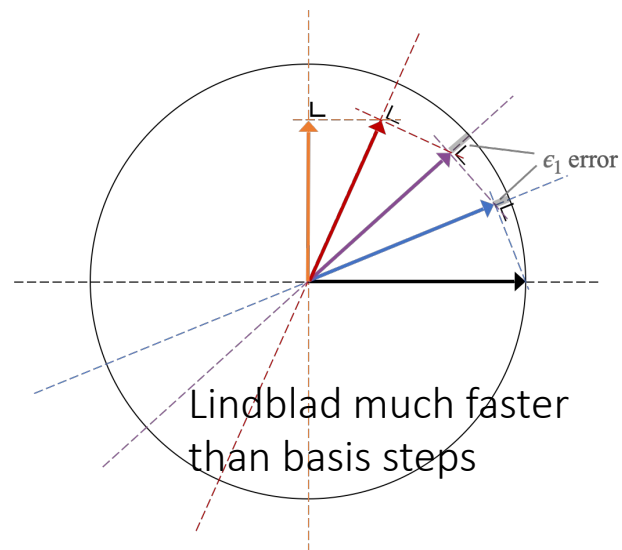
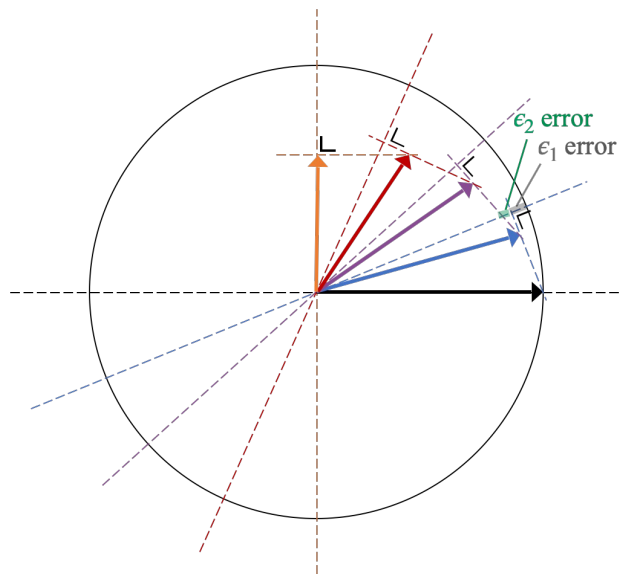
# Straightforward analysis with discrete steps $x(t)$

- Two types of error: - fidelity between old and new basis  
- incomplete projection onto basis



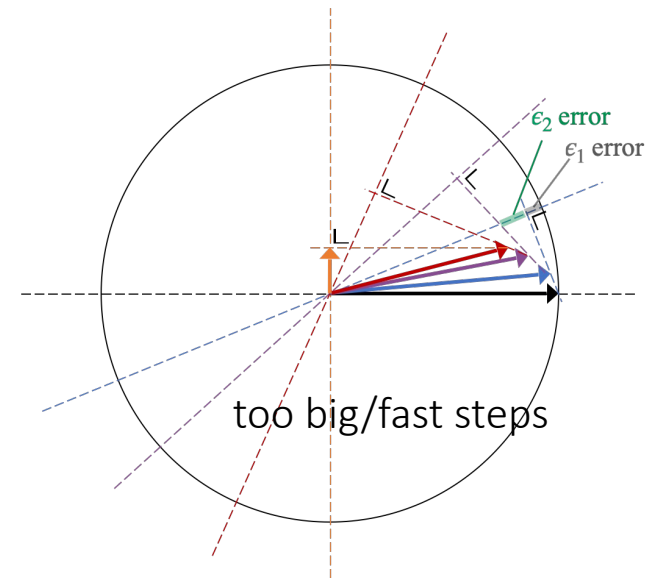
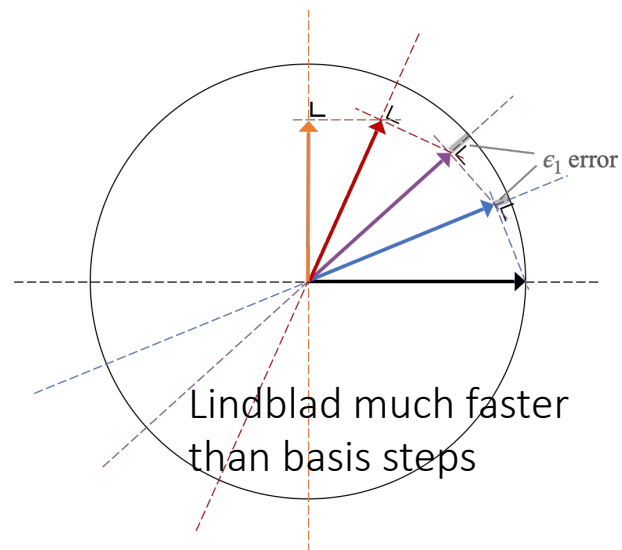
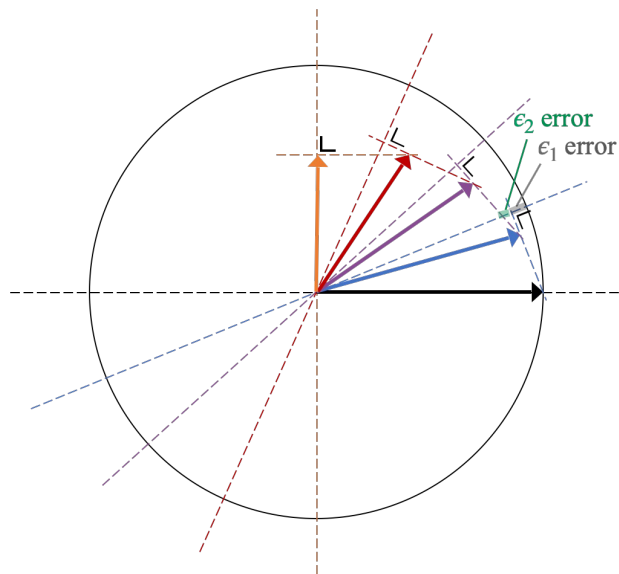
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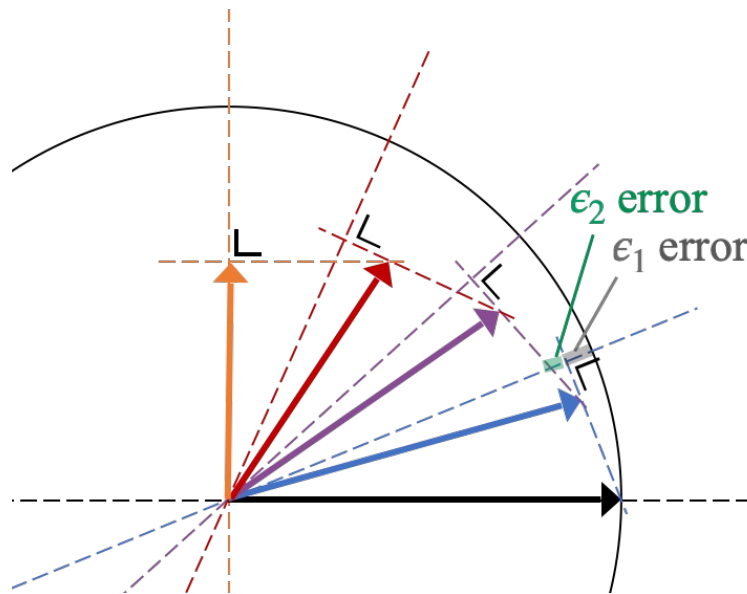
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# Bottleneck: still spectral gap of $H_0$

Lindbladian convergence properties at fixed  $x(s)$  :

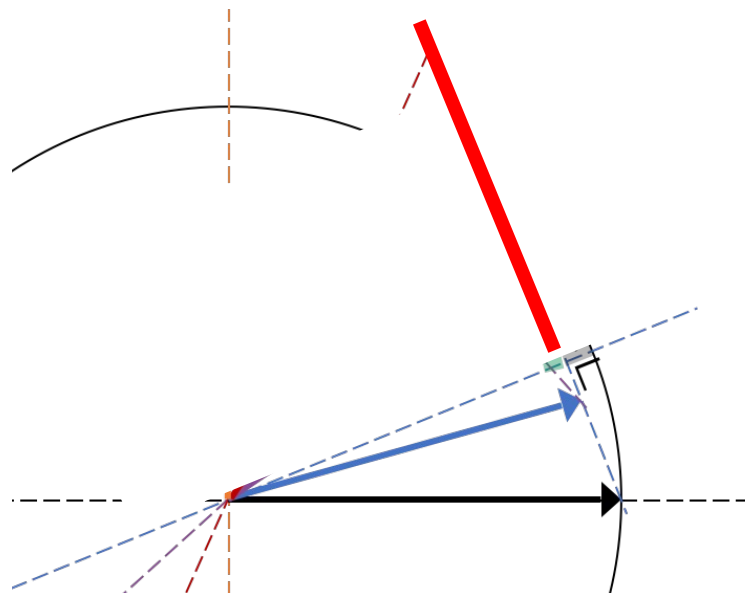
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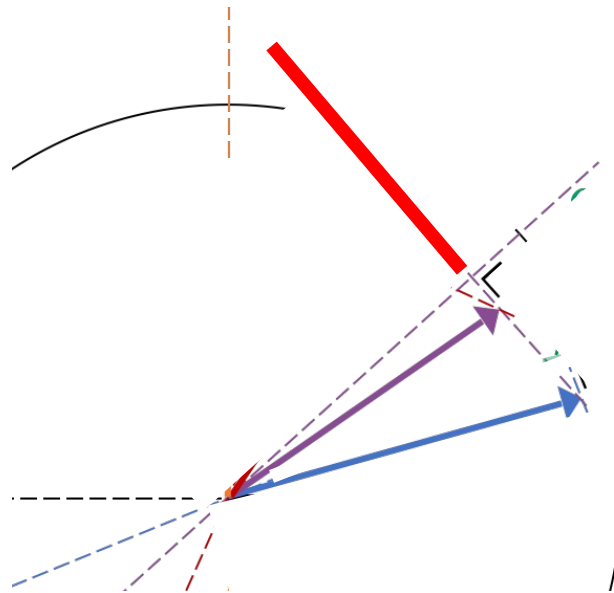
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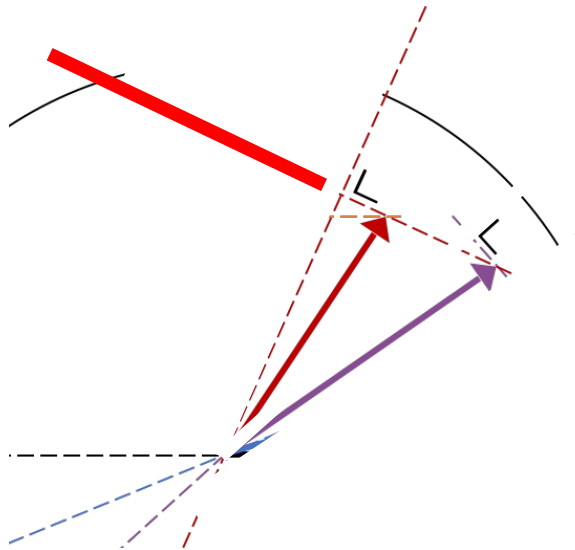
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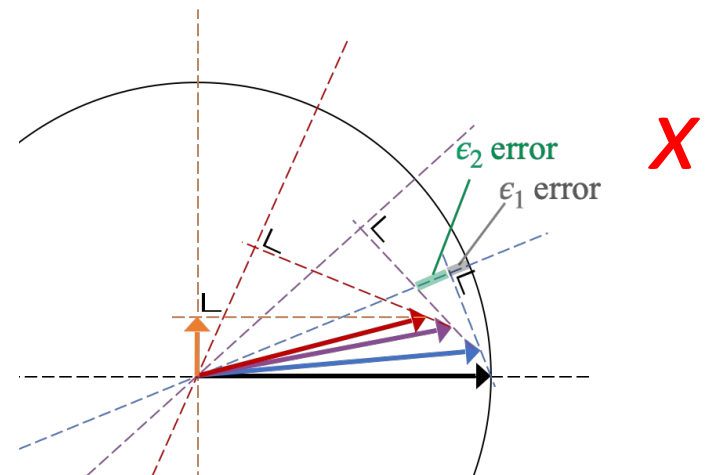
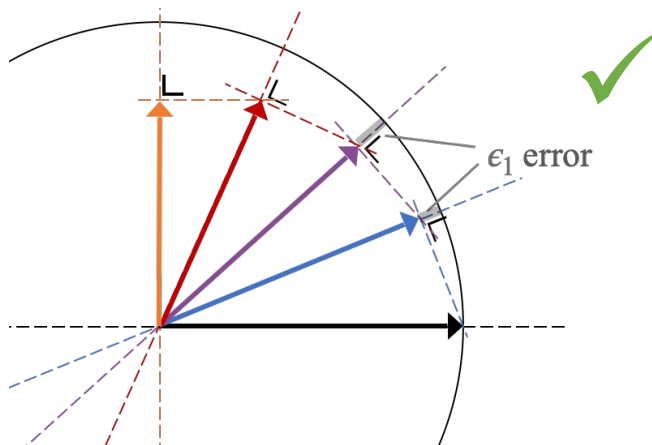
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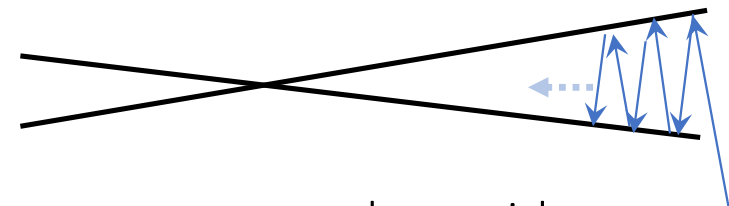


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Lindbladian convergence properties at fixed  $x(s)$  :

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!! decay rate =  $\Delta(s_1)$  **of  $H_0$**



stronger analogy with  
alternated projections method

# Bottleneck: still spectral gap of $H_0$

Lindbladian convergence properties at fixed  $x(s)$  :

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- Task = suppress coherences {solution} vs. {orthogonal Hilbert subspace}

!! decay rate =  $\Delta(s_1)$  **of  $H_0$**

runtime  $T \sim 1/\Delta(t_1)$

« As fast as amplified annealing, but with simpler open quantum system »

# Outline

I. Optimization / classical problem

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Tutorial

II. Quantum annealing methods

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New developments

III. Harnessing quantum dissipation

quantum Zeno dragging

[Last part of the talk still unpublished — end of public slides]

# Future work

- Spectral gap: estimation per k-SAT instance  
is just a bound: go beyond (with Artem Mamichev)
- Clause-by-clause scheduling
- Introducing feedback: algorithmic (e.g. Schoening)  
for Quantum Error Correction / Fault-tolerance

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